(i) **Printed Pages: 3** **Roll No. ....**

(ii) **Questions : 8** **Sub. Code : 0 5 4 2**

Exam. Code : 0 0 0 6

B.A./B.Sc. (General) 6th Semester

(2040)

MATHEMATICS

Paper—II : Linear Algebra

Time Allowed : ~~Three Hours]~~ **[Maximum Marks : 30**

Note: Attempt 50% of Total Questions of Question Paper. Time: 2 Hour
All will carry equal marks. Fraction will be lower digit.

~~questions carry equal marks.~~

SECTION—A

1. (a) Let $V = \{(a, b) : a, b \in \mathbb{R}\}$. Check if V is a vector space over $\mathbb{R}$ or not with addition and scalar multiplication defined as $(a_1, b_1) + (a_2, b_2) = (0, b_1 + b_2)$ and $\alpha(a_1, b_1) = (\alpha a_1, \alpha b_1)$.

   (b) If W_1 and W_2 are any two subspaces of a vector space V(F), prove that $W_1 + W_2 = \{x + y : x \in W_1, y \in W_2\}$ is subspace of V(F) and $W_1 + W_2 = \{W_1 \cup W_2\}$.

   3,3

2. (a) Let V be a vector space over the field F. Prove that the set S of non-zero vector $v_1, v_2, ... v_n \in V$ is linearly dependent iff some of these vectors, say v_k, $2 \le k \le n$, can be expressed as the linear combination of the preceding vectors of the set S.

   (b) Let $V_3(\mathbb{R})$ be a vector space. Check if the set $W = \{(a, b, c) : a = b - c, 2a + 3b - c = 0\}$ is a subspace of $V_3(\mathbb{R})$ or not ? 3,3

3. (a) Find a basis and dimension of the subspace W of $\mathbb{R}^4$ generated by the vectors (1, –4, –2, 1), (1, –3, –1, 2), (3, –8, –2, 7). Also extend these to a basis of $\mathbb{R}^4$.

(b) Prove that the set $B = \{v_1, v_2, ... v_n\}$ is a basis of a finite dimensional vector space V(F) iff each vector $x \in V$ is uniquely expressible as linear combination of the vectors of B. 3,3

4. (a) State and prove Rank Nullity Theorem.

(b) Find a linear map $T : \mathbb{R}^3 \to \mathbb{R}^3$ whose range space is generated by (1, 2, 3) and (4, 5, 6). 3,3

SECTION—B

5. (a) Let $B = \{v_1, v_2, ... v_n\}$ be basis of vector space V(F) and T be a linear transformation on V. Then prove that for any vector $v \in V$, $[T; B][v; B] = [T(v); B]$.

(b) If the matrix of all linear transformation T on $\mathbb{R}^2$ relative to usual basis of $\mathbb{R}^2$ is :

$$\begin{bmatrix} 2 & -3 \\ 1 & 1 \end{bmatrix}$$

Find the matrix of T relative to basis $B_1 = \{(1, 1), (1, -1)\}$. 3,3

6. (a) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $T(x, y) = (5x - y, 8x + 3y)$. Verify Cayley-Hamilton theorem.

(b) Let T be a linear operator on a finite dimensional vector space V. Prove that the following statements are equivalent :

(i) λ is an eigen value of T

(ii) The operator $T - \lambda I$ is singular

(iii) $\text{Det}(T - \lambda I) = 0$. 3,3

7. (a) Let $T : V \to V$ be a linear operator, prove that distinct eigen vectors of T corresponding to distinct eigen values of T are linearly independent.

(b) Prove that every eigen value of a linear operator T on an n-dimensional vector space V(F), is a zero of the minimal polynomial m(x) of T. 3,3

8. (a) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be defined by $T(x, y, z) = (5x - 6y - 6z, -x + 4y + 2z, 3x - 6y - 4z)$. Find minimal polynomial of T and hence find T^{-1}.

(b) Prove that the eigen values of a triangular matrix are just the diagonal entries of the matrix. 3,3