(i) **Printed Pages : 3** **Roll No.** ....

(ii) **Questions : 8**

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Exam. Code : | 0 | 0 | 0 | 6 |

B.A./B.Sc. (General) 6th Semester

(2040)

MATHEMATICS

Paper–I : Analysis–II

~~**Time Allowed : Three Hours]**~~ **[Maximum Marks : 30**

Note: Attempt 50% of Total Questions of Question Paper. Time: 2 Hours All will carry equal marks. Fraction will be lower digit.

SECTION—A

1. (a) Let $T = \{(x, y) : 0 \le x \le 2, 0 \le y \le 3\}$ and $f : T \to \mathbb{R}$ be defined as $f(x, y) = \begin{cases} x & \text{if } 0 \le y \le \frac{2}{3} \\ 0 & \text{if otherwise} \end{cases}$.

Evaluate $\iint\limits_T f(x, y)\, dxdy$.

(b) Evaluate $\iint \sqrt{a^2 - x^2 - y^2}\, dxdy$ over the circle $x^2 + y^2 \le ax$ in the positive quadrant where $a > 0$. 3+3=6

2. (a) Change the order of integration and evaluate the integral

$$\int_0^{4a} \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} dydx.$$

(b) Find the volume of a truncated cone with end radii 'a' and 'b' and height 'h'. 3+3=6

3. (a) State and prove Green's Theorem in a plane.

(b) Verify Stoke's Theorem for $\vec{F} = y\hat{i} + z\hat{j} + x\hat{k}$ where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary. 3+3=6

4. (a) Verify Divergence Theorem for

$$\vec{F} = (x^2 - yz)\hat{i} + (y^2 - zx)\hat{j} + (z^2 - xy)\hat{k}$$

taken over the rectangular parallelopiped $0 \le x \le a$, $0 \le y \le b$, $0 \le z \le c$.

(b) Evaluate $\iint_S \vec{f}\cdot\hat{n}ds$ where $\vec{f} = (x + y^2)\hat{i} - 2x\hat{j} + 2yz\hat{k}$ and S is the surface of the plane $2x + y + 2z = 6$ in first octant. 3+3=6

SECTION—B

5. (a) Prove that a sequence of functions $\{f_n\}$ defined on E converges uniformly on set E iff for every $\varepsilon > 0$ and for all $x \in E$ $\exists$ a +ve integer N such that

$$\left|f_{n+p}(x) - f_n(x)\right| < \varepsilon \ \forall\ n \ge N,\ p \ge 1.$$

(b) Show that the sequence $\{f_n(x)\}$ where $f_n(x) = x^n$ is uniformly convergent on $[0, k]$, $k < 1$ and only pointwise convergent on $[0, 1]$. 3+3=6

6. (a) Discuss for uniform convergence of the series

$$\sum_{n=1}^{\infty}\left[\frac{nx}{1+n^2x^2} - \frac{(n-1)x}{1+(n-1)^2x^2}\right] \text{ in } [0, 1].$$

(b) Let $\{f_n\}$ be a sequence of real valued functions defined on [a, b] and bounded on [a, b] and let $f_n \in R[a, b]$ for n = 1, 2, 3, ... If $\{f_n(x)\}$ converges uniformly to the function f on [a, b] then prove that $f \in R[a, b]$ and

$\int_a^b f(x)\,dx = \underset{n\to\infty}{\text{Lt}} \int_a^b f_n(x)\,dx$. Here R[a, b] denotes set of Riemann integrable functions on [a, b]. 3+3=6

7. (a) Obtain the Fourier Series in the interval $\left[-\frac{1}{2}, \frac{1}{2}\right]$ of the function f(x) given by

$$f(x) = \begin{cases} x - [x] - \frac{1}{2} & \text{if } x \text{ is not an integer} \\ 0 & \text{if } x \text{ is an integer} \end{cases}$$

where [x] is the greatest integer $\le x$.

(b) Find a Fourier series to represent $x - x^2$ from $x = -\pi$ to $x = \pi$. Hence show that $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + ... = \frac{\pi^2}{12}$. 3+3=6

8. Show that :

(i) $\tan^{-1}x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + ...$ for $-1 \le x \le 1$

(ii) $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + ...$

(iii) $\frac{1}{2}(\tan^{-1}x)^2 = \frac{x^2}{2} - \frac{x^4}{4}\left(1+\frac{1}{3}\right) + \frac{x^6}{6}\left(1+\frac{1}{3}+\frac{1}{5}\right) + ...$

where $-1 < x \le 1$. 2+1+3=6